Analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field

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Abstract:
An unsteady viscous incompressible free convective flow of an electrically conducting fluid between two heated parallel plates horizontally is considered under the action of a magnetic field applied transversely to the flow. The induce field is produced along the lines of motion which varies transversely to the flow and the fluid temperature changing with time. Analytical solutions for velocity, induced magnetic field, the temperature distributions and heat transfer are obtained. Graphical results for the velocity distribution of the fluid, the magnetic field and the rate of heat transfer are illustrated and discussed for various values of non-dimensional physical parameters. The skin friction on the two plates are also obtained and plotted. It has been observed that the skin-friction first increases then gradually decreases with the increase of M, the Hartmann number at \( y = \pm 1 \).

Keywords: Induced magnetic field, incompressible, conducting fluid, skin friction, Hartmann number.

1 Introduction

Yang and Yu [1] studied the problem of convective magnetohydrodynamic channel flow between two parallel plates subjected simultaneously to an axial temperature gradient and a pressure gradient numerically. In their conclusion they have found that an applied transverse magnetic field may reduce the entrance length of the velocity considerably, but has little effect on the temperature development. Seth and Ghosh [2] considered the unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid in a rotating channel under the influence of a periodic pressure gradient and of uniform magnetic field, which is inclined with the axis of rotation. An analytical solution to the problem of steady and unsteady hydromagnetic flow of viscous incompressible electrically conducting fluid

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under the influence of constant and periodic pressure gradient in presence of inclined magnetic field has been obtained exactly by Ghosh [3] to study the effect of slowly rotating systems with low frequency of oscillation when the conductivity of the fluid is low and the applied magnetic field is weak. Borkakati and Chakrabarty [4] investigated the unsteady free convection MHD flow between two heated vertical parallel plates in induced magnetic field. Angel, Pop and Hossain [5] studied the heat and mass transfer problem on mixed convection flow over a horizontal plate. The problem of combined heat and mass transfer of an electrically conducting fluid in MHD free convection adjacent to a vertical surface with Ohmic heating and viscous dissipation is analyzed by Chen [6]. He presented the results for the velocity, temperature, and concentration distributions, as well as the local skin-friction coefficient, local Nusselt number, and the local Sherwood number. Chamkha [7] considered the problem of unsteady, two-dimensional, laminar, boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal and concentration buoyancy effects. He solved the problem analytically using two-term harmonic and non-harmonic functions. MHD free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate was investigated by Raptis and Kafousias [8]. Das, Aziz and Ahmed [9] studied free convective steady flow and heat transfer in a viscous incompressible fluid bounded between vertical wavy walls with equal transpiration. Flow of viscous incompressible fluid past semi-infinite parallel plates with variable viscosity was considered by Deka [10]. Aydin and Avci [11] investigated analytically to predict laminar heat convection in a Couette–Poiseuille flow between two parallel plates with a simultaneous pressure gradient and an axial movement of the upper plate. The effect of the modified Brinkman number on the temperature distribution and the Nusselt number has been discussed for different values of the relative velocity of the upper plate.

In the present work, we are presenting our investigations on the effect of transversely applied external magnetic field on the unsteady laminar flow of an incompressible viscous electrically conducting fluid in a channel of two horizontal heated plates. This induces a field along the lines of motion which varies transversely to the flow. The temperature of the fluid motion is assumed to be changing with time. The analytical solutions for the fluid velocity, induced magnetic field, the temperature distribution and the heat transfer are obtained. The velocity distribution of fluid, the magnetic field and the skin frictions at the two plates are obtained for different magnetic field parameters and are plotted graphically. The rates of heat transfer are also obtained and are plotted.

2 Formulation of the Problem

We are considering an unsteady laminar convective flow of a viscous incompressible electrically conducting fluid through two infinite parallel plates channel separated by a distance of \(2h\) horizontally.

Let \(X\)-axis be taken along the horizontal direction through the central line of the channel and \(Y\)-axis is taken perpendicular to the \(X\)-axis. The plates of the channel are at \(y = \pm h\). The uniform magnetic field \(B_0\) is applied parallel to \(Y\)-axis which in turn induces a field along \(X\)-axis that varies along \(Y\)-axis. The velocity and magnetic
field distributions are \( \vec{V} = [u, v, w] = [u(y, t), 0, 0] \) and \( \vec{B} = [B_x, B_y, B_z] = [B(y), B_0, 0] \) respectively. Here \( B_0 \) and \( B(y) \) are applied and induced magnetic field respectively.

Figure 1:

To write down the governing equations of the problem, we are to assume that the fluid under consideration is finitely conducting so that the viscous dissipation and the Joule heat due to the presence of external magnetic field are neglected. Hall effect, Polarization effect and the effect due to Buoyancy are negligible.

Initially (i.e. at time \( t = 0 \)) the plates and the fluid are at zero temperature (i.e. \( T = 0 \)) and there is no flow within the channel.

At time \( t > 0 \), the temperature of the plate \( (y = \pm h) \) change according to \( T = T_0(1 - e^{-nt}) \), where \( T_0 \) is a constant temperature and \( n \geq 0 \) is a real number, denoting the decay factor.

The plates are considered to be infinite and all the physical quantities are functions of \( y \) and \( t \) only.

3 Governing Equations:

Under the above assumptions the governing equations are as follows:

\[
\nabla \cdot \vec{V} = 0 \quad (3.1)
\]

\[
\rho \left( \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right) = -\nabla p + \mu \nabla^2 \vec{V} + (\vec{J} \times \vec{B}) \quad (3.2)
\]

\[
\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{V} \times \vec{B}) - \left( \frac{1}{\sigma \mu_e} \right) \nabla^2 \vec{B} = 0 \quad (3.3)
\]

\[
\rho C_p \left( \frac{\partial T}{\partial t} \right) = k \frac{\partial^2 T}{\partial y^2} + \rho \nu \left( \frac{\partial u}{\partial y} \right)^2 \quad (3.4)
\]

where the third term in the right hand side of equation (3.2) is the magnetic body force and \( \vec{J} \) is the current density due to the magnetic field defined by

\[
\vec{J} = \frac{(\nabla \times \vec{B})}{\mu_e} \quad (3.5)
\]
Where,
\( k \) = Thermal conductivity,
\( \sigma \) = Electrical conductivity,
\( \rho \) = Fluid density,
\( \mu_e \) = Permeability of the medium,
\( \mu \) = Co-efficient of viscosity,
\( \nu_m = \frac{1}{\sigma \mu_e} \), Magnetic diffusivity,
\( \nu = \frac{\mu}{\rho} \), Kinematic viscosity,
\( C_p \) is the specific heat at constant pressure and
\( \beta \) is the coefficient of thermal expansion.

Using the velocity and magnetic field distribution as stated above, the equation (3.1) to equation (3.4) are as follows:

\[
\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{B_0}{(\rho \mu_e)} \frac{\partial B}{\partial y} \tag{3.6}
\]

\[
\frac{\partial B}{\partial t} - B_0 \frac{\partial u}{\partial y} - \left( \frac{1}{\sigma \mu_e} \right) \frac{\partial^2 B}{\partial y^2} = 0 \tag{3.7}
\]

\[
\frac{\partial T}{\partial t} = \left( \frac{k}{\rho C_p} \right) \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2
\]

\[
\Rightarrow \frac{\partial T}{\partial t} = \alpha_1 \frac{\partial^2 T}{\partial y^2} + \alpha_2 \left( \frac{\partial u}{\partial y} \right)^2 \tag{3.8}
\]

Where, \( \alpha_1 = \frac{k}{\rho C_p} \), \( \alpha_2 = \frac{\mu}{\rho C_p} \)

The boundary conditions are

\[
\begin{align*}
\text{at} \ t &= 0, u = 0, B = B_0, T = 0 \at y = \pm h \\
\text{at} \ t > 0, u = u_0, B = B_0 + B_i = B', T = T_0(1 - e^{-nt}) \at y = \pm h
\end{align*} \tag{3.9}
\]

Considering the non-dimensional terms

\[
t^* = \frac{\nu t}{h^2}, \quad b = \frac{B}{B_0}, \quad y^* = \frac{y}{h}, \quad u^* = \frac{u}{u_0}, \quad \text{where} \quad u_0 = \frac{(\beta g T_0 h^2)}{\nu}, \quad \bar{T} = \frac{(T_0 - T)}{T_0}
\]

\[
(3.10)
\]

In terms of the above non-dimensional variables and parameters, the basic equations (3.6)-(3.8) take the form

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \left( \frac{M^2}{R_e R_m P_r} \right) \frac{\partial b}{\partial y} \tag{3.11}
\]

\[
\frac{\partial b}{\partial t} - R_e \frac{\partial u}{\partial y} - \left( \frac{1}{R_m P_r} \right) \frac{\partial^2 b}{\partial y^2} = 0 \tag{3.12}
\]

\[
\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} - E_c \left( \frac{\partial u}{\partial y} \right)^2 \tag{3.13}
\]

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The asterisks have been dropped with the understanding that all the quantities are now dimensionless.

where,

\[ M = \sqrt{\frac{B_0^2 h^2 \sigma}{\rho \nu}}, \]

\[ P_r = \frac{\nu}{\alpha_1}, \]

\[ E_c = \frac{v \alpha_1}{C_p T_0}, \]

\[ R_e = \frac{\nu h}{P_r}, \]

\[ R_a = \frac{\beta gh^3 T_0}{v \alpha_1}, \]

\[ R_m = \frac{\alpha \mu e \sigma}{\nu h}, \]

\[ \alpha_1 = \frac{k}{\rho C_p}, \]

\[ \nu_e = \frac{1}{\sigma \mu e}, \]

\[ \nu = \frac{\mu}{\rho}, \]

\[ \sigma is the electrical conductivity, \]

\[ \rho is the fluid density, \]

\[ \mu_e is the permeability of the medium and \]

\[ \mu is the co-efficient of viscosity, \]

The boundary condition (3.10) reduces to

\[
\begin{align*}
  t = 0 & , u = 0 , b = 1 , T = 1 \text{ at } y = \pm h \\
  t > 0 & , u = 1 , b = B', T = e^{-nt} \text{ at } y = \pm h
\end{align*}
\]

(3.14)

4 Solutions

To solve the equation (3.11) to equation (3.13) subject to the boundary condition (3.14), we apply the transformation of variables

\[ u = f(y)e^{-nt}, \quad b = g(y)e^{-nt} \text{ and } T = \phi(y)e^{-2nt} \]

(4.1)

Substituting (4.1) in Equations (3.11), (3.12), (3.13), We have

From (3.11)

\[
\begin{align*}
  -nf(y)e^{-nt} &= e^{-nt}\frac{\partial^2 f}{\partial y^2} + \left( \frac{M^2}{R_e R_m P_r} \right) \frac{\partial g}{\partial y} e^{-nt} \\
  \Rightarrow \frac{\partial^2 f}{\partial y^2} + nf + \left( \frac{M^2}{R_e R_m P_r} \right) \frac{\partial g}{\partial y} &= 0
\end{align*}
\]

(4.2)

From Eq. (3.12)

\[
\begin{align*}
  -ne^{-nt}g(y) - R_e \frac{\partial f}{\partial y} e^{-nt} - \left( \frac{1}{R_m P_r} \right) \frac{\partial^2 g}{\partial y^2} e^{-nt} &= 0 \\
  \Rightarrow \frac{\partial^2 g}{\partial y^2} + (n R_m P_r)g + (R_e R_m P_r) \frac{\partial f}{\partial y} &= 0
\end{align*}
\]

(4.3)
From Eq. (3.13)

$$-2ne^{-2nt}f(y) = \frac{1}{P_r} \frac{\partial^2 \phi}{\partial y^2} e^{-2nt} - Ec \left( \frac{\partial f}{\partial y} e^{-nt} \right)^2$$

$$\Rightarrow \frac{\partial^2 \phi}{\partial y^2} + (2nP_r) \phi - EcP_r \left( \frac{\partial f}{\partial y} \right)^2 = 0$$  \hspace{1cm} (4.4)

The corresponding boundary conditions are:

for \( t = 0 \) : \( f = 0 \), \( g = 1 \), \( \phi = 1 \) at \( y = \pm 1 \)
for \( t > 0 \) : \( f = e^{nt} \), \( g = \left( \frac{B'}{B_0} \right) e^{nt} \), \( \phi = e^{nt} \) at \( y = \pm 1 \)  \hspace{1cm} (4.5)

The solutions of equations (4.2), (4.3), (4.4) subject to the boundary conditions (4.5) are

$$f[y] = C_1 e^{-\alpha y} + C_2 e^{\alpha y} + C_3 e^{-\beta y} + C_4 e^{\beta y}$$  \hspace{1cm} (4.6)

$$g[y] = e^{-k_0 y}(e^{k_0 y} \sin[\sqrt{k_3} y]C_5 + e^{k_0 y} \cos[\sqrt{k_3} y]C_6 +$$

$$k_4 k_8 (e^{\alpha y} \beta C_3 k_{10} - e^{(\alpha + 2\beta) y} \beta C_4 k_{10} +$$

$$a(e^{\beta y} C_1 - e^{-k_{12} C_1} k_{11}))$$  \hspace{1cm} (4.7)

$$\phi[y] = C_7 \sin[k_1 y] + C_8 \cos[k_1 y] + \left[ \frac{e^{-2\alpha y} \alpha^2 C_1^2}{4\alpha^2 + k_1^2} - \frac{e^{2\alpha y} \alpha^2 C_2^2}{4\alpha^2 + k_1^2} +$$

$$2\alpha C_1 \left( \frac{\alpha C_2^2}{k_1^2} + \frac{e^{(\beta - \alpha)y} \beta C_4}{(\alpha - \beta)^2 + k_1^2} - \frac{e^{-(\alpha + \beta)y} \beta C_3}{(\alpha + \beta)^2 + k_1^2} \right) + \beta \left( \frac{e^{-2\beta y} \beta C_3^2}{4\beta^2 + k_1^2} +$$

$$C_3 \left( \frac{2\beta C_4}{k_1^2} + \frac{2e^{(\alpha - \beta)y} \alpha C_2}{(\alpha - \beta)^2 + k_1^2} \right) + C_4 \left( \frac{e^{2\beta y} \beta C_4}{4\beta^2 + k_1^2} - \frac{2e^{(\alpha + \beta)y} \alpha C_2}{(\alpha + \beta)^2 + k_1^2} \right) \right] k_5$$  \hspace{1cm} (4.8)

where \( \alpha = \sqrt{-k_0 - \sqrt{k_0^2 - 4k_7}} \); \( \beta = \sqrt{-k_0 + \sqrt{k_0^2 - 4k_7}} \);

$$A_1 = \frac{\alpha^2}{4\alpha^2 + k_1}; A_2 = \frac{1}{(\alpha - \beta)^2 + k_1}; A_3 = \frac{1}{(\alpha + \beta)^2 + k_1}; A_4 = \frac{\beta}{4\beta^2 + k_1};$$

$$k_1 = \sqrt{2nP_r}; k_2 = \frac{M^2}{Re R_m P_r}; k_3 = nR_m P_r; k_4 = R_e R_m P_r; k_5 = -Ec P_r;$$

$$k_6 = 2 + k_3 - k_2 k_4; k_7 = n k_3; k_8 = \frac{1}{(\alpha^2 + k_3)(\beta^2 + k_3)}; k_9 = \alpha + \beta;$$

$$k_{10} = \alpha^2 + k_3; k_{11} = \beta^2 + k_3; S_1 = k_2 \sqrt{k_3} \cos\left[ \sqrt{k_3} \right];$$

$$S_2 = k_2 \sqrt{k_3} \sin\left[ \sqrt{k_3} \right];$$

$$S_3 = e^{-\alpha - \beta - k_{12}} \left( e^{\alpha + 3\beta + k_{12}} \beta^2 + e^{2\beta + k_9 + k_{12}} \beta k_2 k_4 k_8 (\alpha - k_9) k_{10} \right);$$

$$S_4 = e^{-\alpha - \beta - k_{12}} \left( e^{\alpha + \beta + k_{12}} \beta^2 + e^{k_9 + k_{12}} \beta k_2 k_4 k_8 (-\alpha - 2\beta + k_9) k_{10} \right);$$
\[S_5 = e^{-\alpha - 3\beta - k_{12}} \left(e^{2\alpha + 2\beta + k_{12}} + e^{\alpha + \beta + k_{12} + k_{24}}k_{4}k_{8}(\beta - k_{9}k_{11})\right)\]

\[S_6 = e^{-\alpha - 2\beta - k_{12}} \left(e^{2\beta + k_{12}} + e^{\alpha + 2\beta + k_{9}}k_{2}k_{4}k_{8}(k_{9} - k_{12})\right);\]

\[S_7 = (e^{-\beta - k_{12}^2} + e^{\alpha - k_{9}}k_{2}k_{4}k_{8}(\alpha - k_{9}k_{10});\]

\[S_8 = (e^{\beta - k_{12}^2} - e^{\alpha + 2\beta - k_{9}}k_{2}k_{4}k_{8}(\alpha + 2\beta - k_{9}k_{10});\]

\[S_9 = (e^{-\alpha - 2\beta - k_{12}} + e^{\beta - k_{9}}k_{2}k_{4}k_{8}(\beta - k_{9}k_{11});\]

\[S_{10} = (e^{\alpha - 2\beta + k_{12}} + e^{\alpha - k_{9}}k_{2}k_{4}k_{8}(k_{9} - k_{12});\]

\[S_{11} = \sin \left(\sqrt{k}_3\right); S_{12} = \cos \left(\sqrt{k}_3\right); S_{13} = e^{-\alpha - k_{9}}k_{2}k_{4}k_{8}k_{10};\]

\[S_{14} = e^{-\alpha - 2\beta + k_{9}}k_{4}k_{8}k_{10}; S_{15} = e^{-\alpha - k_{9}}k_{4}k_{8}k_{11}; S_{16} = e^{\alpha - k_{9}}k_{4}k_{8}k_{12};\]

\[S_{17} = e^{\alpha - k_{9}}k_{4}k_{8}k_{10}; S_{18} = e^{\alpha - 2\beta - k_{9}}k_{4}k_{8}k_{10}; S_{19} = e^{\beta - k_{9}}k_{4}k_{8}k_{11};\]

\[S_{20} = e^{-\alpha - k_{9}}k_{12}\alpha k_{4}k_{8}k_{11}; S_{21} = S_{22} = \frac{(S_{3} - S_{7})S_{12} - S_{2}(S_{13} + S_{17})}{2S_{12}};\]

\[S_{23} = \frac{(S_{4} - S_{8})S_{12} + S_{2}(S_{14} + S_{18})}{2S_{12}}; S_{24} = \frac{(S_{5}S_{12} - S_{9}S_{12} - S_{2}S_{15} - S_{2}S_{19})}{2S_{12}};\]

\[S_{25} = \frac{(S_{6}S_{12} - S_{9}S_{12} + S_{2}S_{16} + S_{2}S_{20})}{2S_{12}}; S_{26} = \frac{(S_{5}S_{11} + S_{7}S_{11} + S_{1}(S_{13} - S_{17}))}{2S_{1}};\]

\[S_{27} = \frac{(S_{4}S_{11} + S_{8}S_{11} + S_{1}(-S_{14} + S_{18}))}{2S_{1}}; S_{28} = \frac{(S_{5}S_{11} + S_{9}S_{11} + S_{1}S_{15} - S_{1}S_{19})}{2S_{1}};\]

\[S_{29} = \frac{(S_{6}S_{11} + S_{10}S_{11} - S_{1}S_{16} + S_{1}S_{20})}{2S_{1}};\]

\[S_{30} = (-Sinh[\alpha + \beta]S_{25}S_{26} - Sinh[2\alpha]S_{22}S_{27} + Sinh[\alpha - \beta]S_{25}S_{27} + Sinh[\alpha - \beta]S_{22}S_{28} + \]

\[Sinh[2\beta]S_{25}S_{28} + Sinh[\alpha + \beta]S_{22}S_{29} + S_{23}(Sinh[2\alpha]S_{26} - Sinh[\alpha + \beta]S_{28} - Sinh[\alpha - \beta]S_{29})\]

\[-S_{24}(Sinh[\alpha - \beta]S_{26} - Sinh[\alpha + \beta]S_{27} + Sinh[2\beta]S_{29});\]

\[S_{31} = (Sinh[\alpha + \beta]S_{25}S_{26} + Sinh[2\alpha]S_{22}S_{27} - Sinh[\alpha - \beta]S_{25}S_{27} - Sinh[\alpha - \beta]S_{22}S_{28} - \]

\[Sinh[2\beta]S_{25}S_{28} - Sinh[\alpha + \beta]S_{22}S_{29} + S_{23}(-Sinh[2\alpha]S_{26} + Sinh[\alpha + \beta]S_{28} + Sinh[\alpha - \beta]S_{29} + S_{24}(Sinh[\alpha - \beta]S_{26} - Sinh[\alpha + \beta]S_{27} + Sinh[2\beta]S_{29});\]

\[C_{1} = \frac{S_{21}(Sinh[\alpha - \beta]S_{26} - Sinh[\alpha + \beta]S_{27} + Sinh[2\beta]S_{29})}{S_{30}};\]

\[C_{2} = \frac{S_{21}(-Sinh[\alpha + \beta]S_{26} + Sinh[\alpha - \beta]S_{27} + Sinh[2\beta]S_{28})}{S_{31}};\]

\[C_{3} = \frac{S_{21}(-Sinh[2\alpha]S_{27} + Sinh[\alpha - \beta]S_{28} + Sinh[\alpha + \beta]S_{29})}{S_{31}};\]

\[C_{4} = \frac{S_{21}(-Sinh[2\alpha]S_{26} + Sinh[\alpha + \beta]S_{28} + Sinh[\alpha - \beta]S_{29})}{S_{30}};\]
\[ C_5 = - \frac{C_3 S_3 + C_4 S_4 + C_1 S_5 + C_2 S_6 + C_3 S_7 + C_4 S_8 + C_1 S_9 + C_2 S_{10}}{2S_1}; \]
\[ C_6 = \frac{2 - C_3 S_{13} + C_4 S_{14} - C_1 S_{15} + C_2 S_{16} - C_3 S_{17} + C_4 S_{18} + C_1 S_{19} + C_2 S_{20}}{2S_{12}}; \]
\[ C_7 = Co sec[k_1] (Sinh[2\alpha]A_1 (C_2^2 - 1) + \beta (2\alpha Sinh[\alpha - \beta]A_2 (C_1 C_4 - C_2 C_3) + 2\alpha Sinh[\alpha + \beta]A_3 (C_2 C_4 - C_3 C_4) + Sinh[2\beta]A_4 (C_4^2 - C_3^2)) k_5; \]
\[ C_8 = \frac{1}{k_1^2} (Sec[k_1] ((-2 \alpha^2 C_1 C_2 + \beta^2 C_3 C_4) k_5 + k_1^2 (1 + (Cosh[2\alpha]A_1 (1 + C_2^2) + \beta (-2\alpha Cosh[\alpha - \beta]A_2 (C_2 C_3 + C_1 C_4) + 2\alpha Cosh[\alpha + \beta]A_3 (C_1 C_3 + C_2 C_4) + Cosh[2\beta]A_4 (C_3^2 + C_4^2)) k_5 )); \]

5 Skin Friction

The skin friction at the plates \( y = \pm 1 \), is defined as

\[ \tau = - \left( \mu \frac{du}{dy} \right)_{y = \pm 1} \quad (5.1) \]

Substituting the non-dimensional quantities (3.10), we get

\[ \tau = - \left[ \mu \frac{\partial u}{\partial \nu} \frac{\partial u^*}{\partial \nu} \frac{\partial y^*}{\partial y} \right]_{y = \pm 1} \]

\[ \Rightarrow \tau = - \left( \mu \frac{\beta g T_0 h}{\nu} \right) \left[ \frac{\partial u^*}{\partial y^*} \right]_{y = \pm 1} \]

removing the asterisks, we get

\[ \tau = - \left( \mu \frac{\beta g T_0 h}{\nu} \right) \left[ \frac{\partial u}{\partial y} \right]_{y = \pm 1} \quad (5.2) \]

using relation (4.1), we get

\[ \tau = - \left( \mu \frac{\beta g T_0 h}{\nu} \right) \left[ \frac{df}{dy} e^{-nt} \right]_{y = \pm 1} \quad (5.3) \]

6 Heat Transfer

The rate of heat transfer i.e., the heat transfer co-efficient in terms of Nusselt number \( (N_u) \) at the plates is

\[ (N_u) = \left[ \frac{\partial T}{\partial y} \right]_{y = \pm 1} \]

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\[= e^{-2nt} \left[ \frac{\partial \phi}{\partial y} \right]_{y=\pm 1}\]
\[= e^{-2nt} \left\{ e^{\pm 2(\alpha + \beta)} (\pm 2(\alpha + \beta) C_7 C_1 - e^{\pm 2(\alpha + \beta)} \sin[\pm k_1] C_8 C_1 +
2(e^{\pm 2(\alpha + \beta)} (1 - e^{\pm 4\alpha} C_2^2) \alpha A_1 + \beta (\alpha - \beta) A_2 (e^{\pm 3(\alpha + \beta)} C_2 C_3 - e^{\pm (\alpha + 3\beta)} C_1 C_4) -
\pm (\alpha + \beta) \alpha (\alpha + \beta) A_3 (e^{\pm 2(\alpha + \beta)} C_2 C_4 - C_1 C_3) + e^{\pm 2(\alpha + \beta)} A_4 (C_3^2 - e^{\pm 4\beta} C_4^2)) k_5) \right\} \] (6.1)

7 Results ans Discussion

The velocity distributions \(f\) against the distance from the fixed plates \(y\) are plotted at different values of magnetic Hartmann number \((M)\) and magnetic Reynolds number \((R_m)\) in the Fig. 2 and 3. On the basis of same consideration Fig. 4 to Fig. 5 and Fig. 6 to Fig. 7 are plotted. Since the problem involves too many non-dimensional parameters, for the sake of conciseness, we fixed some of the parameters namely \(E_c = 0.2, R_e = 1.0,\
\(P_r = 0.71,\) and \(n = 1.5\) for all the numerical computations and analyzed the effect of other important parameters on the flow.

MATHEMATICA V 5.1 is used as tools for computational process. All these plots have also been done by using MATHEMATICA V 5.1.

The velocity and induced magnetic field distributions are shown in Fig. 2 to Fig. 5. The skin frictions at the plates are shown in Fig. 6 and 7 for different values of \(M\) and \(R_m.\) The heat transfer profiles are shown in Fig. 8 and 9. The results obtained from Fig. 2 to Fig. 9 are as follows:

In Fig. 2 it has been observed that in the first half the velocity gradually increases with the increase of \(M\) but it is decreases in the second half. The velocity profiles are zero at the central plane of the channel.

In Fig. 3 it has been observed that the velocity gradually increases with the increase of \(R_m\) in the first half and decreases in the second half. It is also observed that the values of velocities are negative in the first half whereas in the second half, these are positive.

In Fig. 4 and 5 the induced magnetic field strength are plotted against distance from the plates at point equal distance from the plates and at points on the plates. It has been observed that the induced magnetic field profiles are parabolic in nature and attain their maximum near the middle plane of the channel. In Fig. 4 it is also observed that the induced field strength is continuously decreases with an increase in Hartmann number \(M.\) This indicates that the magnetic field strength can be reduced by an increase in the Hartmann number. But in Fig. 5 it is found that the induced field profiles are increases with the increasing values of magnetic Reynolds number \(R_m.\)

The effect of \(M\) and \(R_m\) on the frictional factor at the plates are almost opposite in nature. From Fig. 6 it is observed that the skin-friction is gradually decreases with the increase of \(M\) but in Fig. 7 it is observed that the skin-friction increases gradually with the increasing values of \(R_m.\)

The effect of \(M\) on the heat transfer at the plates are almost opposite in nature. In Fig. 8 it is observed that the heat transfer gradually decreases with the increasing values of \(M\) when \(y = -1,\) but in Fig. 9 it has been observed that the heat transfer increases when \(M\) increases gradually at \(y = +1.\)
References


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