Thermal radiation effects on hydro-magnetic flow due to an exponentially stretching sheet

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Abstract:
A steady laminar two-dimensional boundary layer flow of a viscous incompressible radiating fluid over an exponentially stretching sheet, in the presence of transverse magnetic field is studied. The non-linear partial differential equations describing the problem under consideration are transformed into a system of ordinary differential equations using similarity transformations. The resultant system is solved by applying Runge-Kutta fourth order method along with shooting technique. The flow phenomenon has been characterized by the thermo physical parameters such as magnetic parameter (M), radiation parameter (R) and Eckert number (E). The effects of these parameters on the fluid velocity, temperature, wall skin friction coefficient and the heat transfer coefficient have been computed and the results are presented graphically and discussed quantitatively.

Key words: Thermal radiation; MHD; Boundary layer flow; exponentially stretching sheet.

1 Introduction
The study of viscous incompressible flow over a stretching surface has become increasing important in the recent years due to its numerous industrial applications such as the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film, condensation process of metallic plate in a cooling bath and glass, and also in polymer industries. Since the pioneering work of Sakiadis [1], various aspects of the stretching flow problem have been investigated by many authors like Cortell [2], Xu and Liao [3], Hayat et al [4] and Hayat and Sajid [5]. On the other hand, Gupta and Gupta [6] stressed that realistically, stretching surface is not necessarily continuous. Due to the fact that the rate of cooling influences the quality of the product with desired characteristics, Ali [7] has investigated the thermal boundary layer flow by considering the nonlinear stretching surface. Further, a new dimension has been added to this investigation by Elbashbeshy [8] who examined the flow and heat transfer characteristics by considering an exponentially stretching continuous surface. He considered an exponential similarity variable and exponential stretching velocity distribution on the coordinate considered in the direction of stretching.
For some industrial applications such as glass production and furnace design, electrical power generation, Astrophysical flows, solar power technology, which operates at high temperatures, radiation effects, can be significant.


There has been a renewed interest in studying magnetohydrodynamic flows and heat transfer due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems involving electrically conductive fluids. In addition, this type of flow finds applications in many engineering problems such as MHD generators, Plasma studies, Nuclear reactors, and Geothermal energy extractions.


In this paper an attempt is made to investigate the effects of thermal radiation on the study laminar two dimensional boundary layer flow of a viscous incompressible electrically conductive and radiating fluid over an exponentially stretching sheet. The governing boundary layer equations are solved using Runge-Kutta fourth order along with shooting technique.

2 Mathematical Formulation

A two dimensional boundary layer flow of a viscous incompressible electrically conductive and radiative fluid bounded by a stretching surface is considered. The x-axis is taken along the stretching surface in the direction of the motion and y-axis perpendicular to it. The fluid is assumed to be gray, absorbing-emitting but non scattering. A uniform magnetic fluid is applied in the direction perpendicular to be stretching surface. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. Then under the above assumptions, in the absence of an input electric field, the governing boundary layer equations are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}
\]

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \tag{2.2}
\]

\[
\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \mu \left( \frac{\partial u}{\partial y} \right)^2 \tag{2.3}
\]

where \(u\) and \(v\) are the velocities in the x- and y-directions respectively, \(\nu\)-the kinematic viscosity, \(\sigma\)-the electrical conductivity, \(B_0\)-the magnetic induction, \(\rho\)-the fluid density, \(c_p\)-the specific heat at constant pressure, \(T\)-the temperature, \(k\)-the thermal conductivity, \(q_r\)-the radiative heat flux and \(\mu\)-the dynamic viscosity.

The second and third terms on the right hand side of Equation (2.3) represent the radiative heat flux and the viscous dissipative heat.

The boundary conditions for the velocity and temperature fields are:

\[
u (0) = U_0 e^\frac{y}{L}, \quad v (0) = 0, \quad T (0) = T_\infty + T_0 e^{\frac{2y}{L}},
\]

\[
u \to 0, \quad T \to 0 \text{ as } y \to \infty \tag{4}
\]
in which $U_0$ - the reference velocity, $T_0$ and $T_\infty$ - the temperatures at far away from the plate and $L$ - the constant. By employed Rosseland approximation (Sajid and Hayat 2008), the radiative heat flux is given by

$$q_r = \frac{4\sigma^* \partial T^4}{3k^* \partial y}$$

(2.4)

where $\sigma^*$ is the Stefan- Boltzmann constant and $k^*$ - the mean absorption coefficient.

We should be noted that the by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences with in the flow field are sufficiently small, then Equation (2.4) can be linearized by expanding $T^4$ into the Taylor series about $T_\infty$, which after neglecting higher order forms takes the form

$$T^4 \approx 4T_\infty^3 T - 3T^4_\infty$$

(2.5)

Invoking Equations (2.3),(2.4) and (2.5), it can be written as

$$\rho c_p \left(\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \left(k + \frac{16\sigma^* T_\infty^3}{3k^*}\right) \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2$$

(2.6)

Introducing the following non-dimensional quantities

$$u = U_0 e^{x L} f' (\eta),

v = -\sqrt{\frac{\nu U_0}{2L}} e^{x L} \left\{ f (\eta) + \eta f' (\eta) \right\} ,

T = T_\infty + T_0 e^{2x L},

\eta = \sqrt{\frac{U_0}{2\nu L}} e^{x L} y, \ M = \frac{\sigma B^2 2L}{\rho U_0 e^{x L}}, \ Pr = \frac{\mu c_p}{k}, \ R = \frac{4\sigma^* T_\infty^3}{k^* k}, \ E = \frac{U_0^2}{T_0 e^{x L}} .$$

(2.7)

Equation (2.1) is automatically satisfied, and Equations (2.2) and (2.6) reduce to

$$f''' - 2(f')^2 + ff'' - Mf' = 0$$

(2.8)

$$\left(1 + \frac{4}{3} R \right) \theta' \ Pr \ (f\theta' - 4f'\theta + E(f'')^2) = 0$$

(2.9)

Where $M, R, Pr, \text{and} E$ are the magnetic parameter, radiation parameter, Prandtl number and Eckert number, respectively and primes denote the differentiation with respect to $\eta$.

The corresponding boundary conditions are

$$f(0) = 0, \ f'(0) = 1, \ \theta(0) = 1, \ f' \rightarrow 0, \ \theta \rightarrow 0 \ as \ \eta \rightarrow \infty$$

(11)

For the type of boundary layer flow, the skin-friction coefficient and heat transfer coefficient are important physical parameters.

Knowing the velocity field, the skin-friction at the stretching surface can be obtained, which in non-dimensional form is given by

$$c_f = -2(Re)^{-\frac{1}{2}} f''(0)$$

(2.10)

Knowing the temperature field, the rate of heat transfer coefficient at the stretching surface can be obtained, which in non-dimensional form, in terms of the Nusselt number, is given by

$$Nu = -(Re)^{\frac{1}{2}} \theta'(0)$$

(2.11)

Where $Re = \frac{U_0 k^*}{\nu}$ is the Reynolds number.
3 Solution of the problem

The governing boundary layer Equations (2.8) and (2.9) subject to boundary conditions (11) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all higher order non-linear differential Equations (2.8) and (2.9) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et al. [16]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta \eta = 0.05$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient and the Nusselt number which are respectively proportional to $f''(0)$ and $-\theta'(0)$, are also sorted out and their numerical values are presented in a tabular form.

4 Results and Discussion

In order to get a physical insight of the problem, a representative set of numerical results we shown graphically in Figs.1-12 to illustrate the influence of physical parameters viz., the magnetic parameter $M$, Prandtl number $Pr$, Radiation parameter $R$ and Eckert number $E$ on the velocity $f'(\eta)$, $f(\eta)$ and temperature $\theta(\eta)$.

Fig.1 depicts the profiles of velocity, $f(\eta)$ and $\theta(\eta)$ profile for $M=1$, $Pr = 1$, $R = 1$ and $E = 0.2$. It is observed that the profiles of the velocity $f'(\eta)$ and $f(\eta)$ are inversely proportional
to each other. The velocity profile is unique for all values of $M, Pr, R$ and $E$ due to the decoupled Equations (2.8) and (2.9).

Figs. (2.2) and (2.3) illustrate the velocity and temperature profiles for different values of the magnetic parameter $M$. It is observed that the velocity decreases as the magnetic parameter increases (Fig. 2). This is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. From Fig. (2.3), it is noticed that an increase in the magnetic parameter results in an increase in the temperature.

The effect of the Prandtl number $Pr$ on the temperature field is shown in Fig. 4. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. It is noticed that as $Pr$ increases, the temperature decreases. This is because, physically, if $Pr$ increases, the thermal diffusivity decreases and these phenomena lead to the decreasing of energy ability that reduces to thermal boundary layer.

The influence of the thermal radiation parameter $R$ on the temperature is shown in Fig. 5. The radiation parameter $R$ defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is observed that as $R$ increases, the temperature profiles and thermal boundary layer thickness also increase.

For different values of the viscous dissipation parameter i.e., the Eckert number $E$ on the temperature is shown in Fig. 6. The Eckert number $E$ express the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conservation of kinetic energy into internal
energy by work done against the viscous fluid stress. The positive Eckert number implies cooling of the sheet i.e., loss of heat from the sheet to the fluid. It is found that the temperature profiles and thermal boundary layer thickness increase slightly with an increase in E.

For further observations, comparison is made between the various physical parameters involved in the problem and shown in Figs.7-10. The effects of R and E with fixed M = 1 and Pr = 1 are shown in Fig.7. It is seen that as E or R increases, the temperature profiles also increase and the effects of R are more pronounced than the effects of E. The effects of E and Pr, with fixed M = 1 and R = 1 are illustrated in Fig. 8. It is observed that their effects are opposite in nature, in which the increase in E and the decrease in Pr lead to the increase in the temperature profiles. The effects of M and Pr, with fixed E = 0.5 and R = 1 are shown in Fig. 9. It is found that their effects are opposite in nature, in which the increase in M and the decrease in Pr lead to the increase in the temperature profiles. The effects of R and M with fixed E = 0.5 and Pr = 1 are presented in Fig.10. It is noticed that as M and R increase, the temperature profiles also increase and the effects of R are more pronounced than the effects of M.

From equations (2.8) and (2.10), it is clear that the variations in the Prandtl number Pr, radiation parameter R and Eckert number E do not effect the wall skin-friction coefficient due to the decoupled equations. However, since the magnetic parameter M is coupled with the momentum equation, it has significant effect on the wall skin-friction coefficient. The wall skin-friction coefficient has unique values 1.28213 and 1.62918, for non-magnetic (M=0) and magnetic (M=1) cases respectively. It is interesting to note that the value of the wall skin-friction coefficient in non-magnetic case is in good agreement with that of Bidin and Nazar [12], whose solved the problem using the Keller-box method.

The effects of various governing parameters on the Nusselt number Nu are shown in Table 1. It is observed that the Nusselt number increases as Pr increases, where as it decreases as M or R or

<table>
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<td>0.5</td>
<td>0.5</td>
<td>3.0</td>
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Table 1: Nu for various values of M, R, E and Pr
E increases.

References


