Combined effects of thermal radiation and heat generation on natural convection in a square cavity filled with Darcy-Forchheimer porous medium

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\textbf{ABSTRACT}

This paper investigates the combined effects of thermal radiation and heat generation on natural convection in a square cavity filled with porous medium using finite-difference technique in staggered grid distribution. In the present study, the cavity is assumed to have isothermal vertical walls and adiabatic horizontal walls. The hydrodynamic field in the porous medium is modelled according to the general model involving Brinkman and Forchheimer terms. Here, parametric study for a wide range of Rayleigh number ($Ra$), Darcy number ($Da$), thermal radiation parameter ($NR$), heat generation parameter ($He$) is done, which shows consistent performance of the numerical approach used for obtaining the solutions as streamlines, isotherms, velocity profiles, temperature profiles, local Nusselt numbers and the average Nusselt numbers. For the purpose of numerical simulation, $Pr = 1$ and $\epsilon = 0.4$ are considered in this work. The results are also computed for vertical velocity and temperature profiles for non-porous case by taking $\epsilon = 1.0$ and $Da = 10^4$. Heat transfer rates at the heated walls are presented in terms of local Nusselt number. The effect of increasing the thermal radiation parameter is to enhance the vertical velocity. Average Nusselt number increases with increase in the thermal radiation parameter whereas reverse effect is observed in the case of heat generating parameter increase.

\textbf{Keywords:} Natural convection; heat transfer; Darcy-Forchheimer porous medium; thermal radiation; heat generation.

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1. Introduction

Natural convection flow in a cavity filled with a porous medium has received considerable attention because of its application to the thermal performance of many engineering installations. Porous materials are often found in nature as well as in many industrial applications. In this context, non-Darcy effects on natural convection in porous media have received a great deal of attention in the recent times due to a large number of technical applications associated with it such as fluid flow in geothermal reservoirs, separation processes in chemical industries, solidification of casting, thermal insulation, petroleum reservoir, and so on. Many works on this subject [1–5] had used simplification based on Darcy’s model, on the assumption of constant porosity medium when the permeability of the porous medium is high, then Darcy’s model does not yield results as satisfactory as those obtained by the experimental results found by [6–8]. Many models including Darcy’s proposition with Forchheimer’s and Brinkman’s extensions are found [9–14]. The model developed by Nithiarasu et al. [14] with Forchheimer’s and Brinkman’s terms was called the generalized model. Medeiros et al. [15] studied heat transfer by natural convection in a porous cavity under a non-Darcian approach for uniform porosity using Darcy-Brinkman-Forchheimer model numerically (finite-volume method). They compared the numerical results with the works that considered

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uniform porosity and found good accuracy. Most recently, Basak et al. [16] have investigated natural-convection flow in a square cavity filled with a porous medium for both uniform and non-uniform heating from below by using the Darcy-Forchheimer model. Some studies on natural-convection flow in a porous cavity with/without heat generation are also reported in the literature [17–21]. In obtaining numerical solution of Navier-Stokes equations, based on primitive variable approach, determination of pressure poses the major problem for incompressible flows. Pressure is virtually decoupled from continuity equation. In MAC (marker-cell), proposed by Harlow and Welch [22], pressure equation was derived from momentum and continuity equations. Some well-known computer codes in staggered grid were developed by Patankar and Spalding [23], Patankar [24], Amsden and Harlow [25], Hirt et al. [26], Van Doormal and Raithby [27]. Marcondes et al. [28] studied the effects of variable porosity on the heat transfer by natural convection, in a cavity with isothermal vertical walls and adiabatic horizontal walls and a porous medium inside.

It is well known that the effect of thermal radiation is important in space technology and high temperature processes. Thermal radiation also plays an important role in controlling heat transfer process in polymer processing industry. Recent developments in hypersonic flights, missile technology and gas cooled nuclear reactors have focussed the attention of the researcher on thermal radiation as a mode of energy transfer and emphasize the need of inclusion of radiative transfer in these processes. Natural convection in fluid saturated heat generation porous media has many applications in science and technology. Such applications include heat transfer associated with the deep storage of nuclear waste, exothermic reactions in packed bed reactors, flow past heat exchanger tubes etc. Detailed literature survey on mentioned topic has shown that these two new dimensions can be added on analyzing natural convection flow in a cavity filled with a porous medium for studying the effects of thermal radiation and internal heat source/sink. Because of the ever increasing scientific and technological applications, the effects of thermal radiation and heat generation have become very important in the mentioned fields of fluid dynamics which demand extensive research work. The effect of radiation on heat transfer problems have been studied by Hossain and Takhar [29]. Pal and Mondal [30] investigated the effects of radiation on combined convection over a vertical flat plate embedded in a porous medium of variable porosity. In critical technological applications like nuclear reactor cooling, the reactor bed can be modeled as a heat generating porous medium of cylindrical cross-section, quenched by a convection flow [31]. Recently, Reddy and Narasimhan [32] studied the heat generation effects in natural convection inside a porous annulus.

The aim of the present work is to study the influence of thermal radiation and heat generation on natural-convection flow in a square cavity filled with a fluid-saturated porous medium with isothermal vertical walls and adiabatic the horizontal walls by considering Darcy-Brinkman-Forchheimer model. The main equations are solved using the finite-difference technique with staggered grid formulation [34, 35]. To treat the pressure-velocity coupling, Bi-CG-Stab algorithm is used. In the present numerical method, pressure equation is derived by using the continuity equation. The numerical results for streamlines, isotherms, velocity and temperature profiles and the heat transfer rate at the heated walls and average Nusselt numbers are presented graphically.

2. Governing equations and boundary conditions

Natural convection heat transfer in an incompressible fluid in a square cavity of side length $L$ filled with sparsely packed porous medium is considered in the presence of thermal radiation and heat source/sink effects. The RosseLAND approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the $x$-direction is assumed to be negligible in comparison to the $y$-direction. Here the cavity filled with porous medium of uniform porosity has isothermal vertical walls and adiabatic horizontal walls. The geometry of this cavity together with the boundary conditions are illustrated in Figure 1.

In the Cartesian coordinate system, the fundamental governing equations are as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$  (2.1)
The associated boundary conditions are

\[
U(X,0) = U(X, L) = U(0, Y) = U(L, Y) = 0, \quad V(X, 0) = V(X, L) = V(0, Y) = V(L, Y) = 0,
\]

\[
\frac{\partial U}{\partial Y}(X, 0) = \frac{\partial V}{\partial Y}(X, L) = 0, \quad T(0, Y) = T_h \quad \text{and} \quad T(L, Y) = T_c.
\]
Now, introducing dimensionless parameters given as follows:

\begin{equation}
\frac{x}{L} = \frac{Y}{L}, \quad u = \frac{UL}{\alpha}, \quad v = \frac{VL}{\alpha}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad Pr = \frac{\nu}{\alpha}, \quad t = \frac{\alpha t'}{L^2},
\end{equation}

\begin{equation}
p = \frac{PL^2}{\rho \alpha^2}, \quad Da = \frac{K}{L^2}, \quad Ra_a = \frac{\theta}{T_c}, \quad R_a = \frac{\kappa^*}{\nu^2} L^3 P_r, \quad N_r = \frac{\kappa^*}{4\sigma T_c^3}, \quad He = \frac{QL^2}{\kappa}.
\end{equation}

Here \( g, \kappa, C_p \) and \( Q \) denote acceleration due to gravity, thermal conductivity, specific heat at constant pressure and heat generation constant respectively. \( t', T, T_h \) and \( T_c \) indicate time, fluid temperature, temperature of the hot wall and temperature of the cold wall, respectively. Here \( \alpha, \rho, \nu, \mu \) and \( Da \) denote thermal diffusivity, density, kinematic viscosity, dynamic viscosity of the fluid and the Darcy number respectively. \( L \) is an appropriate macroscopic length scale that can be used to relate flow in a porous medium with flow in a clear fluid. \( \epsilon, R_a, N_r \) and \( He \) are the porosity of the porous medium, Rayleigh number, thermal radiation parameter and heat generation/absorption parameter respectively. Here \( K \) denotes permeability of the porous medium and the radiation heat flux \( q_r \) is considered according to Rosseland approximation such that \( q_r = -(4\sigma/3\kappa^*)(\partial T^4/\partial Y) \), where \( \sigma \) and \( \kappa^* \) are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Following Raptis [35], the fluid-phase temperature differences within the flow are assumed to be sufficiently small so that \( T^4 \) may be expressed as a linear function of temperature. This is done by expanding \( T^4 \) in a Taylor series about the free stream temperature \( T_c \) and neglecting higher order terms to yield, \( T^4 = 4T_c^3T - 3T_c^4 \).

Using dimensionless quantities given by equations (2.5)-(2.6), the following dimensionless governing equations are:

\begin{equation}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\end{equation}

\begin{equation}
\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + Pr \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{\epsilon} \left( \frac{\partial u}{\partial x} + \frac{\partial uv}{\partial y} \right) - \frac{P_r \epsilon}{Da} u - \frac{1.75 \sqrt{u^2 + v^2}}{\sqrt{150Da \epsilon}} u,
\end{equation}

\begin{equation}
\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + Pr \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{1}{\epsilon} \left( \frac{\partial v}{\partial y} + \frac{\partial uv}{\partial x} \right) - \frac{P_r \epsilon}{Da} v.
\end{equation}
Figure 7: Streamlines and isotherms for $D_a = 10^{-6}$, $R_a = 10^8$, $Pr = 1$, $N_R = 1$ and $He = 0.2$: (a-b).

Figure 8: Streamlines and isotherms for $D_a = 10^{-4}$, $R_a = 10^8$, $Pr = 1$, $N_R = 1$ and $He = 0.2$: (a-b).

\[ \frac{-1.75\sqrt{u^2 + v^2}}{\sqrt{150Da}} v + Pr Ra \theta, \quad (2.9) \]

\[ \frac{\partial \theta}{\partial t} = \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \left( \frac{\partial u \theta}{\partial x} + \frac{\partial v \theta}{\partial y} \right) + \frac{4}{3N_R} \frac{\partial^2 \theta}{\partial y^2} + He \theta. \quad (2.10) \]

The dimensionless boundary condition are

\[ u = v = \frac{\partial \theta}{\partial y} = 0 \text{ at } y = 0 \quad y = 1, \quad (2.11) \]

\[ u = v = 0 \text{ at } x = 0 \quad x = 1, \quad (2.12) \]

\[ \theta = 1 \text{ at } x = 0 \text{ and } \theta = 0 \text{ at } x = 1. \quad (2.13) \]

The heat transfer coefficient in terms of the local Nusselt number ($Nu$) is $Nu = -\partial \theta / \partial n$, where $n$ denotes the normal direction to a plane. The local Nusselt number at the left vertical wall ($Nu_l$) is $Nu_l = -\partial \theta / \partial x \bigg|_{x=0}$.

The average Nusselt number at the hot wall is $\overline{Nu_H} = \int_0^1 Nu dy$. The relationships between stream function, $\psi$ and velocity components for two-dimensional flows are $u = \partial \psi / \partial y$, $v = -\partial \psi / \partial x$. From this definition of stream function, the positive sign of $\psi$ denotes anti-clockwise circulation and the clockwise circulation is represented by the negative sign of $\psi$.

3. Numerical Solution

3.1 Method of solution

Control-volume based finite-difference discretization of the above equation have carried out in the present work in staggered grid, popularity known as MAC cell. In this type of grid alignment, the velocities and the pressure are
evaluated at different locations of the control volume, the pressure and temperature are evaluated at same locations of control volume as shown in Figure 1(b). The difference equations have been derived in distinct types of cells for the four equations, viz., (i) continuity cell, (ii) u-momentum cell, (iii) v-momentum cell, (iv) temperature cell. These distinct cells have been shown in the Figures 2(a) and 2(b). In the present work, control-volume based finite-difference discretization of the non-dimensional governing equations are carried out in a staggered grid, popularly known as MAC cell. The derivatives involved in convective terms are discretized using a hybrid scheme which is a convex combination of second-order central-difference and second-order upwind difference scheme. But the derivatives involved in other terms are discretized using second-order central-difference scheme. The pressure-Poisson equation has been derived from the discretized momentum and continuity equations. In the derivation of pressure Poisson equation, the divergence term at \( n \)-th time level \( (D^n_{ij}) \) is retained and evaluated in the pressure-Poisson iteration. It is done because the discretized form of divergence of velocity field, i.e., \( D^n_{ij} \) is not guaranteed to be zero. The solution procedure starts with the initialization of the velocity field. This is done either from the result of previous cycle or from the prescribed initial and boundary conditions. Using this velocity field, pressure-Poisson equation has been solved by using Bi-CG-Stab method. Knowing pressure field, the \( u \)-momentum, \( v \)-momentum and temperature equations are solved and the values of \( u, v, \theta \) are updated to get the values at \( (n+1) \)th time level. Using the values of \( u \) and \( v \) at \( (n+1) \)th time level, the value of the divergence of velocity field is checked for its limit. If its absolute value is less than \( 0.5 \times 10^{-5} \) and steady state is attained then iteration process stops, otherwise pressure-Poisson equation has to be solved again for pressure field.

3.2 Numerical Stability Criteria

Linear stability of fluid flow is \( \delta t_1 \leq \text{Min} \left[ \frac{\delta x}{u^n}, \frac{\delta y}{v^n} \right] \), which is related to convection of fluid, i.e., fluid should not move more than one cell width per time step (Courant, Friedrichs and Lewy condition). Again accordingly to Hirt’s stability analysis, \( \delta t_2 \leq \text{Min} \left[ \frac{1}{P_r} \frac{\delta x^2 \delta y^2}{(\delta x + \delta y)^2} \right] \). This condition roughly states that momentum cannot diffuse more than one cell width per time step. The time step actually used in the computations is determined from \( \delta t = \text{FCT} \times \left[ \text{Min}(\delta t_1, \delta t_2) \right] \), where the factor FCT varies from 0.2 to 0.4. The upwinding parameter \( \beta \) is governed by the inequality condition \( 1 \geq \beta \geq \text{Max} \left[ \frac{|u|}{\delta x}, \frac{|v|}{\delta y} \right] \). As a rule of thumb, \( \beta \) is taken approximately 1.2 times larger than what is found from the above inequality condition.

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4. Results and discussions

In this study, numerical results have been obtained for natural convection flow in unit square cavity for grid size (80 × 80) and different Rayleigh number (10^8 ≤ Ra ≤ 5 × 10^9), Darcy number (10^{-6} ≤ Da ≤ 10^{-2}), thermal radiation parameter (0.0 ≤ Nr ≤ 10.0), heat generation parameter (0.0 ≤ He ≤ 10.0) which are illustrated through several Table and Figures. Comparison of the average Nusselt number at the hot wall (x = 0) for a non-Darcian regime with uniform porosity has been made with Nithiarasu et al. [14] and Medeiros et al. [15] for a uniform porosity of the porous medium in Table 1. It is noted from this table that an average Nusselt number increases with increase in the value of Ra.

Figures 3 and 4 show the variation of vertical velocity and temperature profiles, respectively at the mid-horizontal plane of the cavity in the absence of thermal radiation and heat generation parameter for Pr = 1 and Da = 10^{-6} by considering uniform porosity ε = 0.4 for different values of Ra, which varies from 10^8 to 5 × 10^9. The results are compared with those of Marcondes et al. [28] and excellent agreement has been obtained with the present results which show that the present numerical method is perfectly suitable for the type of problem considered in this paper and also validates the present numerical scheme. Also, it is observed from figure 3 that...
walls are to be significant in developing the thermal boundary layer. Also increased, due to the greater magnitudes of the stream functions. Further, it is observed that circulations are very small core region occurs such that the isotherms become almost parallel to the vertical walls. Thus higher can be seen from figures 7 – 9. It is seen from figures 8 and 9 that boundary layers are relatively thicker and a figure that the heat transfer rate increases when Rayleigh number increases. In the case of uniform heating, the absence of thermal radiation and heat generating parameter are displayed in figure 11. It is observed from this figure that the temperature profiles coincide at the mid point of the cavity for all the values of Darcy number.

The effect of $Da$ and $Ra_a$ on the streamlines and isotherms are illustrated in the figures 7-10 when $Pr = 1$, $NR = 1$ and $He = 0.2$ and for different values of $Da = 10^{-6}$ to $10^{-2}$ and $Ra_a = 10^8$ to $10^{10}$ with uniform heating of left vertical wall. Due to heating of the left vertical wall, the fluid rises up along the sides of the hot vertical wall and flows towards the cold vertical wall, forming a roll with clockwise rotation inside the cavity. Figure 7 depicts the streamlines and isotherms for $Da = 10^{-6}$ and $Ra_a = 10^8$ which shows that the flow is very weak and hence the isotherms change very smoothly from the hot vertical wall to the cold vertical wall and certainly the heat transfer is dominated by the conduction mode. When figure 7 is compared with the figures 8 and 9, which are plotted for $Da = 10^{-4}$, $10^{-2}$ and $Ra_a = 10^{8}$, it is observed from these figures that the streamlines and isotherms are concentrated near the edges of left and right vertical walls due to stronger circulation. This is the outcome of higher heat transfer rate due to convection. The fluid circulation is strongly dependent on Darcy number as can be seen from figures 7 – 9. It is seen from figures 8 and 9 that boundary layers are relatively thicker and a very small core region occurs such that the isotherms become almost parallel to the vertical walls. Thus higher permeability of the porous medium is significantly more important for the heat transfer rate. Figure 10 is the plot of streamlines and isotherms for $Da = 10^{-6}$ and $Ra_a = 10^8$. A comparison of figure 7 and figure 10 reveal that as Rayleigh number increases from $10^8$ to $10^9$ with $Da = 10^{-6}$, the buoyancy-driven circulation inside the cavity is also increased, due to the greater magnitudes of the stream functions. Further, it is observed that circulations are greater near the center and least at the wall. Thus, the temperature gradient near both the left and right vertical walls are to be significant in developing the thermal boundary layer.

The effects of Rayleigh number ($10^5 \leq Ra_a \leq 5 \times 10^9$) on the local Nusselt numbers at left vertical wall in the absence of thermal radiation and heat generating parameter are displayed in figure 11. It is observed from this figure that the heat transfer rate increases when Rayleigh number increases. In the case of uniform heating, $Nu_{l}$ is very high at the bottom edge of left vertical wall, due to the presence of discontinuity in the temperature boundary

<table>
<thead>
<tr>
<th>$Ra_a$</th>
<th>Nithiarasu et al.[14]</th>
<th>Medeiros et al.[15]</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^9$</td>
<td>2.97</td>
<td>3.05</td>
<td>3.07</td>
</tr>
<tr>
<td>$10^8$</td>
<td>11.46</td>
<td>12.10</td>
<td>12.23</td>
</tr>
<tr>
<td>$5 \times 10^9$</td>
<td>23.09</td>
<td>24.74</td>
<td>24.43</td>
</tr>
</tbody>
</table>

Table 1: Comparison of average Nusselt number $\overline{Nu}_{H_{l}}|_{x=0}$ of the generalized model in the non-Darcian regime, $Pr = 1$, $Da = 10^{-6}$, $\epsilon = 0.4$, $NR = 0.0$, $He = 0.0$. There is abrupt variation in the vertical velocity profile near the hot and cold walls due to high Rayleigh number i.e $Ra_a = 5 \times 10^9$ and hence the velocity profiles are highly altered by the convection. Figure 4 shows that the temperature decreases when the values of $Ra_a$ increases up to a certain value of $x (= 0.5)$, but beyond that distance the opposite trend is observed.

Figures 5 and 6 show the effect of Darcy number on vertical velocity and temperature profiles at mid-horizontal plane when $Pr = 1$, $Ra_a = 10^9$, $NR = 1$ and $He = 0.2$. From figure 5 it is observed that the vertical velocity at mid-horizontal plane increases with increase in the Darcy number near the hot wall and cold wall of the cavity by keeping $Pr = 1$, $Ra_a = 10^5$, $NR = 1$ and $He = 0.2$. But an abrupt increase in the velocity is seen near the hot wall which forms a very high peak, whereas reverse effect is observed near the cold wall of the cavity in the absence of porous medium (i.e. when $\epsilon = 1$ and $Da = 10^4$) in the mid-horizontal plane. Figure 6 shows that the temperature profiles at mid-horizontal plane decrease in the presence of porous medium when Darcy number increases near the hot wall up to a certain value of $x (= 0.5)$ but beyond that distance the opposite trend is observed near the cold wall of the cavity. Very low temperature is observed in absence of porous medium (i.e. when $\epsilon = 1$ and $Da = 10^{-4}$) at mid-horizontal plane near the hot wall whereas enhanced temperature profile is seen at the cold wall of the cavity. Further, it is observed from this figure that the temperature profiles coincide at the mid point of the cavity for all the values of Darcy number.

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conditions at this edge. Further, the local heat transfer rate at the cold right vertical wall has its minimum value. The effects of thermal radiation on the heat transfer rates are displayed in figure 12. From this figure it is seen that $N_{u\xi}$ increases when $N_R$ increases up-to a certain value of $y (= 0.55)$ but beyond that height the opposite trend is observed. Figure 13 presents the variation of $N_{u\xi}$ for various values of $R_a$ and $He$. It is observed that $N_{u\xi}$ decreases when $He$ increases for both $R_a = 10^8$ and $R_a = 10^9$.

5. Conclusion

The combined effects of thermal radiation and heat generation on natural convection in a square cavity filled with porous medium in the Darcy-Brinkman-Forchheimer model is studied in the present paper. Results for the vertical velocity and temperature at the mid-horizontal plane of the cavity, local Nusselt number and average Nusselt number are obtained for representative governing physical parameters. Streamlines and isotherms for various values of Darcy number and Rayleigh number are shown graphically. As a summary, we conclude the following:

1. The effect of increasing the thermal radiation parameter is to enhance the vertical velocity.
2. The vertical velocity decreases with increasing the value of heat generating parameter on the left vertical wall whereas reverse effect is observed on the right vertical wall.
3. The temperature decreases with increase in the value of Rayleigh number up to certain value of $x$ and beyond that distance the opposite trend is observed.
4. Average Nusselt number increases with increase in the thermal radiation parameter whereas reverse effect is observed in the case of heat generating parameter increase.

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